Brief Report

Network geometry breakdown, not entropy increase: a neurotopological interpretation of epileptic seizures

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Abstract

Epileptic pathology is commonly interpreted as a state of increased neural disorder or high entropy. We challenge this interpretation and propose that such conclusions arise from incomplete or unidimensional metric approaches. Using a unified neurotopological framework, we characterize brain dynamics by integrating both informational content and network geometric structure. We quantify neural coherence through a multimodal metric set including permutation entropy, persistent homology, and Ricci curvature. The framework is validated using EEG data by comparing status epilepticus with healthy resting-state activity (*isostasis*). Our results ($p < 0.001$) show that epileptic seizures do not reflect heightened disorder but instead constitute a catastrophic collapse of network geometry, resulting in disordered low-complexity coherence. Furthermore, we identify a paradox in which the resting healthy brain demonstrates low-ordered complexity, yet remains functionally coherent. These findings suggest that the critical biomarker differentiating health from pathology is not entropy magnitude but the dynamic geometry of neural coherence.

Keywords: *Independent component analysis, EEG, Ricci curvature, persistent homology, epilepsy, isostasis*

1. Introduction

Epilepsy has been described through the lens of hypersynchrony, where the primary pathology is understood as an excess of simultaneous neuronal firing (Engel, 2001). To quantify this, existing literature has extensively employed non-linear analysis methods, specifically various forms of entropy (approximate entropy, sample entropy, and permutation entropy) to detect the transition from interictal to ictal states. However, this approach has led to a significant contradiction in the field: while some studies suggest seizures represent a transition to "chaos" (high entropy), others argue they represent a "locking" of the system (low entropy). This ambiguity indicates that current metrics, which focus primarily on the *quantity* of information or randomness, are insufficient to fully capture the structural alterations of the epileptic brain (Yaffe et al., 2015).

The necessity of resolving this paradox lies in the distinction between information and geometry. Recent advances in network neuroscience suggest that the brain should be modeled not just as a producer of signals, but as a topological space with a specific shape and connectivity structure (Stam, 2014).

While traditional metrics can detect *that* a change has occurred, they often fail to explain *how* the network's architecture has fundamentally reorganized. To address this gap, this paper proposes a shift from purely statistical measures to a unified neurotopological framework. By integrating topological data analysis (Persistent Homology and Ricci Curvature) with information theory, we aim to characterize the seizure not merely as a change in disorder, but as a catastrophic collapse of the network's functional geometry.

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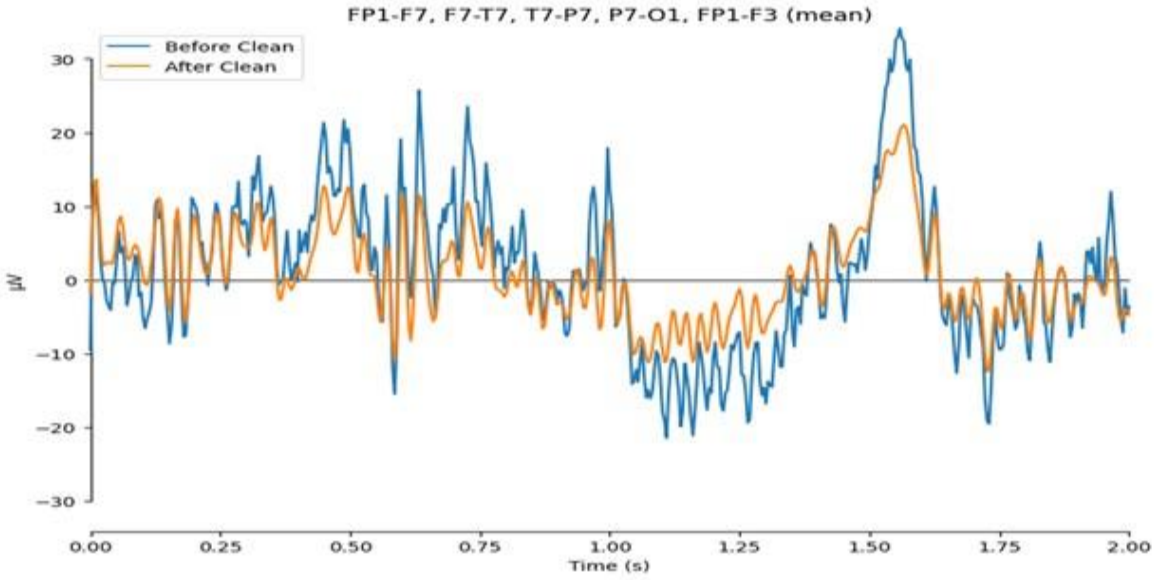

Figure 1 *Comparison of the average evoked EEG signal (patient chb01). The blue line (Before Clean) shows the raw signal with artifacts; the orange line (After Clean) shows the neural signal after ICA artifact removal.*

2. Methods

2.1 Multimetric neurotopological analysis protocol

The methodological framework for this study integrates three interdependent mathematical instruments to evaluate the EEG signal.

Permutation entropy is used to quantify neural signal irregularity, implemented computationally using the AntroPy library (Vallentín,2023). Second, network connectivity is established by Pearson's Correlation Coefficient, converting the strength of the correlation into a Distance Matrix for geometric analysis.

The key metric for local geometric stability is the Ricci curvature. It quantifies the robustness of brain network connections using the Wasserstein distance (optimal transport) between the probability distributions of the nodes, implemented through SciPy (Virtanen et al.,2020).

A negative curvature suggests stiffness and collapse, while a positive curvature suggests flexibility and coherence. Additionally, we used Persistent Homology, a Topological Data Analysis (TDA) technique (Reimann et al.,2017), to quantify the overall shape of the network.

The geometrical and informational metrics are defined as (Stam.,2014):

Entropy $H = -\sum p_i \log(p_i)$; *Correlation* $r = \frac{Cov(X,Y)}{(\sigma_x \sigma\gamma)}$; *Distance* $D = 1 - |r|$; *Ricci curvature* $K(u, v) = 1 - W(u, v)$.

The Permutation entropy (H): $H = -\sum p_i \log(p_i)$, where p_i is the probability distribution of the signal values, quantifying signal complexity; the Correlation (r): $r = \frac{Cov(X,Y)}{(\sigma_x \sigma\gamma)}$, representing the Pearson's correlation coefficient; the Distance (D): $D = 1 - |r|$, used to convert correlation strength into a distance matrix for geometric analysis; and the Ricci curvature (K): $K(u, v) = 1 - W(u, v)$, where $W(u, v)$ represents the Wasserstein distance (optimal transport) between probability distributions of adjacent nodes, used to quantify local network geometry.

2.2 Data acquisition and preparation

Empirical data for validation were obtained from the public CHB-MIT Scalp EEG database (PhysioNet) (Goldberger et al., 2000), an established resource for complex physiological signals containing EEG recordings of pediatric subjects with epilepsy (patients

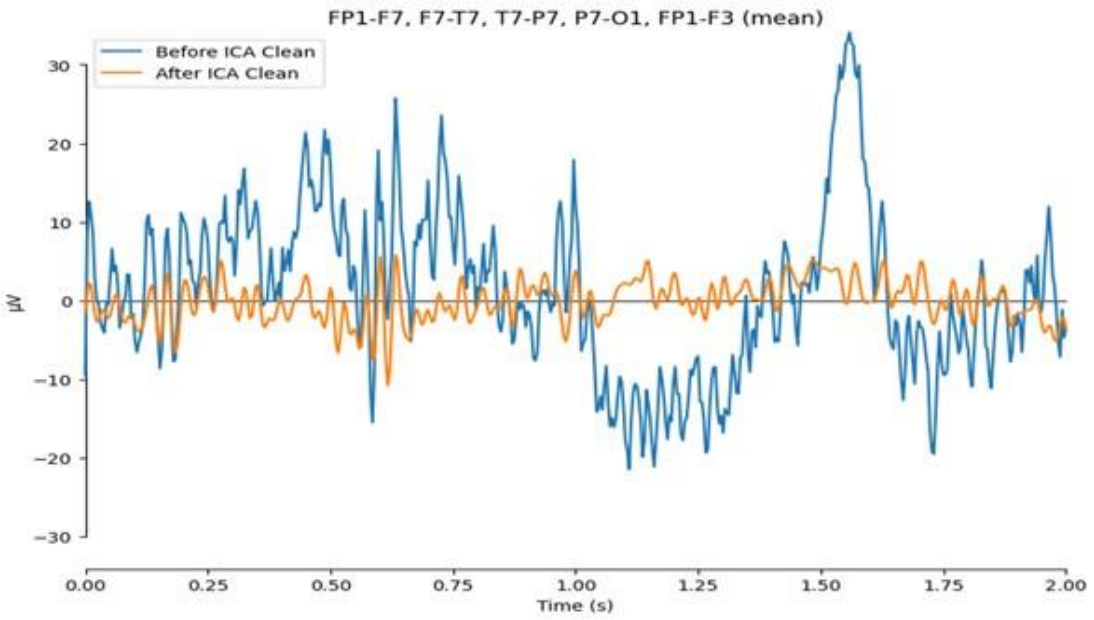

Figure 2 *Topography of Independent Component Analysis (ICA). This map illustrates the isolation of artifactual sources (e.g., blink artifacts in frontal zones) effectively removed to reconstruct the clean EEG data (X_{clean}).*

1-4). These were compared with a healthy control at rest (isostasis).

Raw EEG data (***X***) are mixtures of independent sources (***S***), including brain activity and artifacts (e.g., eye movement). Independent Component Analysis (ICA) was used to isolate these sources by estimating a demixing matrix (***W***) and filtering was managed using the ICA/MNE-Python package (Gramfort et al.,2013):

$$\mathbf{S} = \mathbf{W} \cdot \mathbf{X}$$

In this expression, the bold typeface indicates a matrix or vector quantity, ***X*** is the observed signal matrix (the raw EEG data, a mixture of sources), ***S*** is the independent source matrix (the separated components of brain activity), ***W*** is the estimated demixing matrix, and the dot (·) denotes matrix multiplication.

After estimation, the artifactual components are identified and removed. The clean EEG data ($\boldsymbol{S}_{\mathbf{clean}}\ \boldsymbol{X}_{\mathbf{clean}}$) is reconstructed using the reverse mixing matrix:

$$\boldsymbol{X}_{\mathbf{clean}} = \mathbf{A} \cdot \boldsymbol{S}_{\mathbf{clean}}$$

where ***A*** is the mixing matrix (the inverse of ***W***) used to reconstruct the cleaned signal $\boldsymbol{X}_{\boldsymbol{clean}}$ from the non-artifactual source components $\boldsymbol{S}_{\boldsymbol{clean}}$.

This procedure effectively removes non-neuronal artifacts while preserving the true neural signal for analysis. **Figs. 1** and **2** illustrate the result of this cleansing process.

The analysis was designed in two phases. First, an internal comparison of epileptic patients was performed to validate the ability of the multimetric mark to differentiate pathological phases, even when traditional metrics fail. Second, neurotopological signatures were compared in three critical phases: ictal (during the crisis), pre-ictal (before), and post-ictal (after).

3. Results

The presents the results of the Kolmogorov-Smirnov (KS) test and the p-values to differentiate the three critical phases of the epileptic seizure are summarized in **Table 1**. The results demonstrate that traditional metrics, such as PermEntropy or Fractal Dimension (HiguchiFD), are not sufficient to meaningfully distinguish ($p > 0.05$) between phases. On the other hand, the Ricci curvature proves to be a robust marker, showing statistically significant differences ($p < 0.05$)

between all phases, validating its superiority to characterize the collapse geometry. Although patient's brains differ, Ricci curvature proved to be a robust marker, showing significant differences ($p < 0.05$) between all phases, while Permutation Entropy alone was not sufficient ($p > 0.05$) (see **Figs. 3 and 4)**.

The unique ictal signature is further demonstrated by temporal and spectral measures, highlighting the chaotic but individual nature of the event **(see Figs. 5 and 6)**.

Table 1 *Statistical comparison of neurotopological metrics across seizure phases (Pre-ictal, Ictal, Post-ictal).*

Comparison	PermEntropy_KS	Perm Entropy_p	PermEntropy_Interpretation
Before vs During	0.14493	>0.05	Not Significant
Before vs After	0.17391	>0.05	Not Significant
During vs After	0.08696	>0.05	Not Significant

HiguchiFD_KS	HiguchiFD_p	HiguchiFD_Interpretation
0.17391	>0.05	Not Significant
0.17391	>0.05	Not Significant
0.05797	>0.05	Not Significant

HurstExp_KS	HurstExp_p	HurstExp_Interpretation
0.21739	>0.05	Not Significant
0.07246	>0.05	Not Significant
0.24638	>0.05	Significant

LZComplexity_KS	LZComplexity_p	LZComplexity
1	>0.05	Significant
0	>0.05	Not Significant
1	>0.05	Significant

RicciCurvature_KS	RicciCurvature_p	RicciCurvature_p
0.36232	>0.05	Significant
0.47826	>0.05	Significant
0.27536	>0.05	Significant

Computational analysis of the EEG data revealed fundamental neurotopological distinctions between epileptic subjects and healthy control. The application of an analysis of variance (ANOVA) and Kolmogorov-Smirnov (K-S) tests the metrics demonstrated a statistically significant difference in the average values of Network Entropy and Ricci curvature that was consistent in all phases analyzed (Pre-Ictal, Ictal and Post-Ictal), resulting in a *p-value* < 0.001. This initial finding suggests that the neurotopological signature of the epileptic brain is basally distinct from that of the healthy brain, and not a simple momentary aberration of the seizure. The Wasserstein distance (optimal transport) between the probability distributions of the nodes, a negative value suggests rigidity and collapse, while a positive value suggests flexibility and coherence. Persistent homology (Reimann et al.,2017) (not parameterized here) is further used to quantify topological shape and distances (bottleneck).

The nature of this statistical difference reveals the central empirical paradox. Contrary to "high entropy" theories, our results identify ictal crisis as a catastrophic collapse into disordered low complexity. This phase was characterized by information annihilation (critically low entropy), structural rigidity (markedly negative Ricci curvature), and complete topological breakdown, evidenced by a high Bottleneck distance in H1 homology when compared to the healthy control.

Crucially, the neurotopological signature of healthy resting control (isostasis) was also presented as a state of low entropy. However, their topology was fundamentally opposite, manifesting as a low ordered complexity. This state was characterized by a coherent and flexible architecture (positive and stable Ricci curvature) and an intact homological structure. The Kolmogorov-Smirnov test (*whose p-value* ≈ 1.0) suggested that, although the forms of entropy distribution could be similar between states, their averages ($p<0.001$) were in opposite domains, validating the qualitative distinction between geometric order and disorder.

The integration of these metrics within the unified neurotopological framework reveals a clear geometric separation between healthy and pathological conditions, as visually demonstrated in the comparative analysis **(Fig.8)**.

The complete results of these statistical comparisons, detailing the P-Values of the analysis of variance, the results of the K-S test and the Bottleneck Distances for the three phases (Pre-Ictal, Ictal, Post-Ictal) against control (*homeostasis/isostasis*), are summarized in **Table 2**. The Kolmogorov-Smirnov (K-S) test results and p-values comparing the Healthy Control against the epileptic patient across three phases (Before, During, After). The results show statistically significant differences ($p < 0.001$) in both Permutation entropy

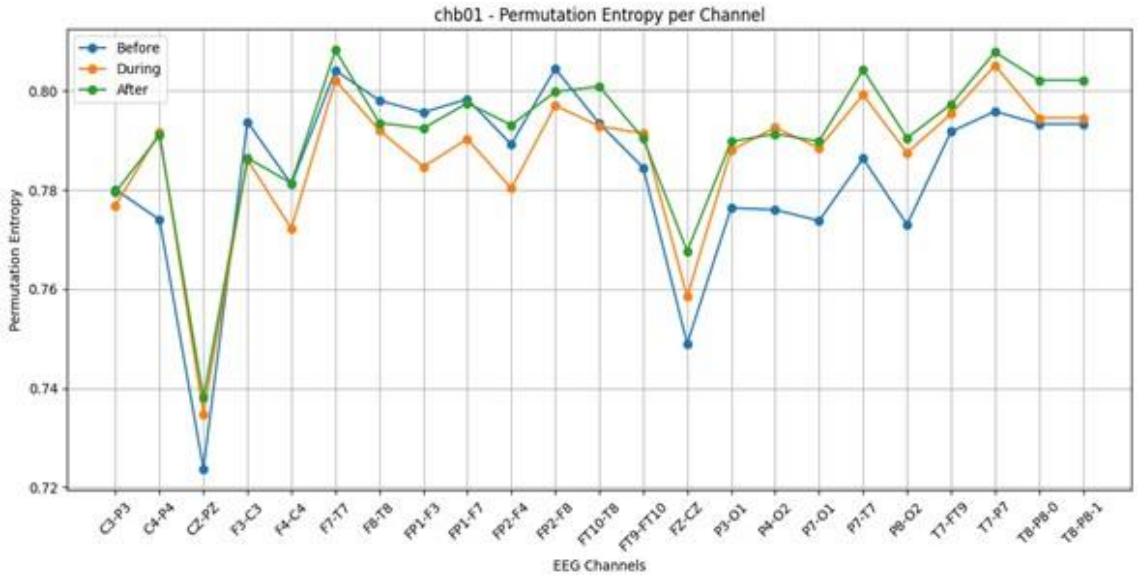

Figure 3 *Permutation entropy (H) per EEG channel across seizure phases. The overlap between phases (Before/During/After) explains the lack of statistical significance for this metric alone.*

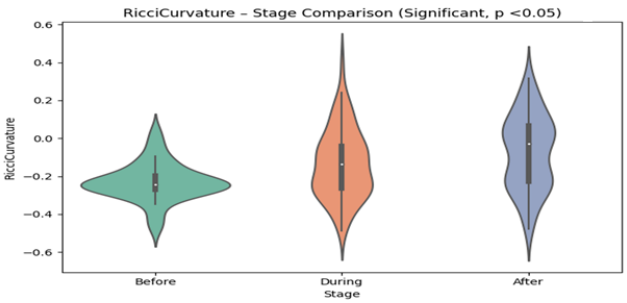

Figure 4 *Ricci curvature analysis. The graph demonstrates how geometric curvature changes, indicating a loss of communication efficiency (negative values) during the seizure collapse.*

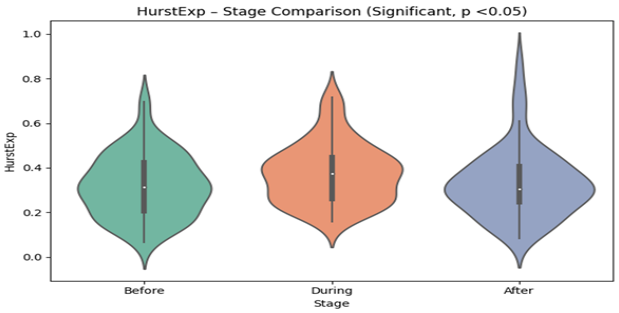

Figure 5 *Hurst exponent distribution. Comparing the "memory" of the signal across phases, showing significant alterations in temporal dependence during the ictal state.*

and Ricci curvature for all comparisons. This confirms that the neurotopological signature of the healthy brain (isostasis) is fundamentally distinct from the epileptic brain, even during non-seizure phases.

4. Discussion

This study, we compared brain network topology at several preictal intervals (3 days, 3 hours, 5 minutes, and 1 minute before seizure), as well as during ictal and postictal phases. Results showed that epileptic patients consistently had higher neural complexity than healthy controls. According to Fractal–Basal Dynamic, this rise indicates a pre-collapse instability — where excessive synchronization and loss of flexibility in neural networks lead to seizure onset — contrasting with the stable, self-regulated topology of healthy brains.

The central contribution of this work is the resolution of a fundamental empirical paradox. Our results demonstrate that while both healthy control at rest and the patient during ictal crisis may exhibit low-entropy states, their neurodynamic geometries are topologically opposed. As evidenced in our analyses (**Table 1**), traditional entropy metrics alone are insufficient to distinguish these phases ($p > 0.05$). In contrast, our geometric framework, (see **Table 2**), successfully differentiates these states with high statistical significance ($p < 0.001$).

Table 2 *Statistical comparison of neurotopological signatures between epileptic patients and healthy control.*

Comparison	PermEntropy_KS	PermEntropy_p	PermEntropy _Interpretation	RicciCurvature_KS	RicciCurvature_p	RicciCurvature_ Interpretation
Healthy vs Before	1	p < 0.001	Extremely Significant Difference	0.97101	p < 0.001	Extremely Significant Difference
Healthy vs During	1	p < 0.001	Extremely Significant Difference	0.7805	p < 0.001	Extremely Significant Difference
Healthy vs After	0.99123	p < 0.001	Extremely Significant Difference	0.56381	p < 0.001	Extremely Significant Difference

Table 3 *Qualitative and quantitative interpretation of neurodynamic differences.*

Metric	Patient Value	Healthy Value	Scientific Interpretation
Kurtosis	3.55 (near normal)	197.16 (very sharp)	The healthy brain shows high-energy peaks (spikes) during cognitive activity, while the patient dis plays a flat energy distribution.
Permutation entropy (H)	0.78	1.57	Strong reduction in temporal complexity → Collapse of temporal dynamics, a core indicator.
Higuchi Fractal dimension (D)	1.57	0.06	The difference is numerically inverted, but generally the patient's signal is more linear and less hierarchically structured.
Lempel–Ziv Complexity (LZC)	0.0019	0.0009	Both are low due to short signals, but the patient's slightly higher value reflects irregular, unstructured randomness.
Hurst exponent (H _exp)	0.34	0.65	The healthy signal shows long-range temporal dependence, while the patient's activity reflects short-memory, unstable states.
Ricci curvature (K)	In	1.0	Missing curvature suggests loss of topological stability in the patient's neural network; the healthy brain maintains coherent geometry.
Coherence (Coh)	In	0.45	The patient's brain loses inter-channel synchronization, indicating degraded functional integration.

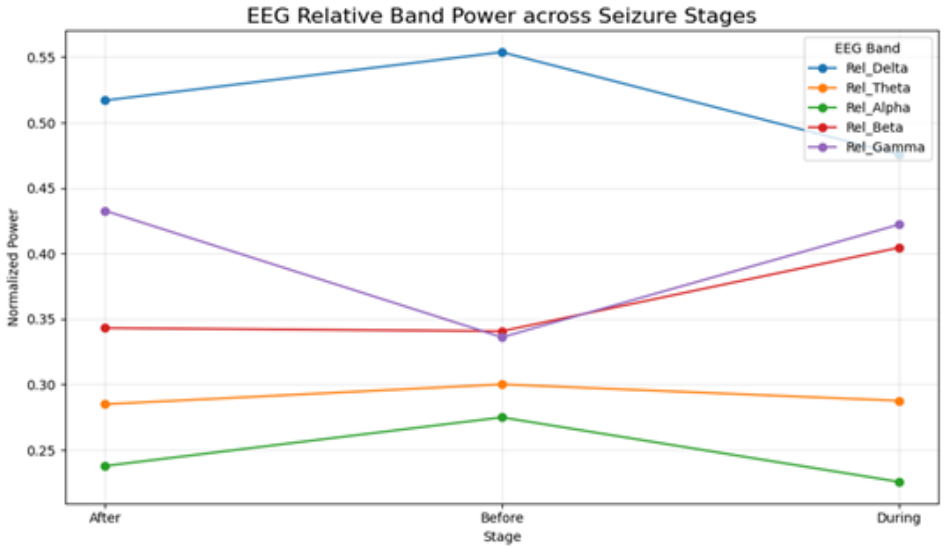

Figure 6 *EEG relative band power. The graph illustrates the surge in Delta/Beta waves and the suppression of alpha waves, a hallmark of the ictal state.*

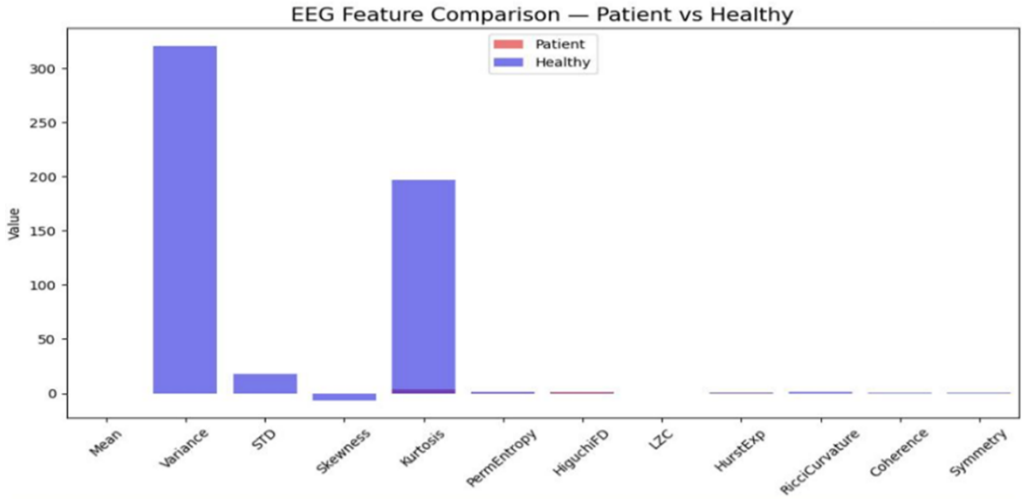

Figure 7 *EEG Feature comparison: Patient vs. Healthy Control. This bar chart contrasts key neurodynamic features between the epileptic patient (red) and the healthy control (blue). Notably, the healthy brain exhibits significantly higher Variance and Kurtosis, reflecting a dynamic and responsive range. In contrast, the patient displays flattened values, indicating a signal with reduced dynamic variability and a "collapsed" distribution structure*

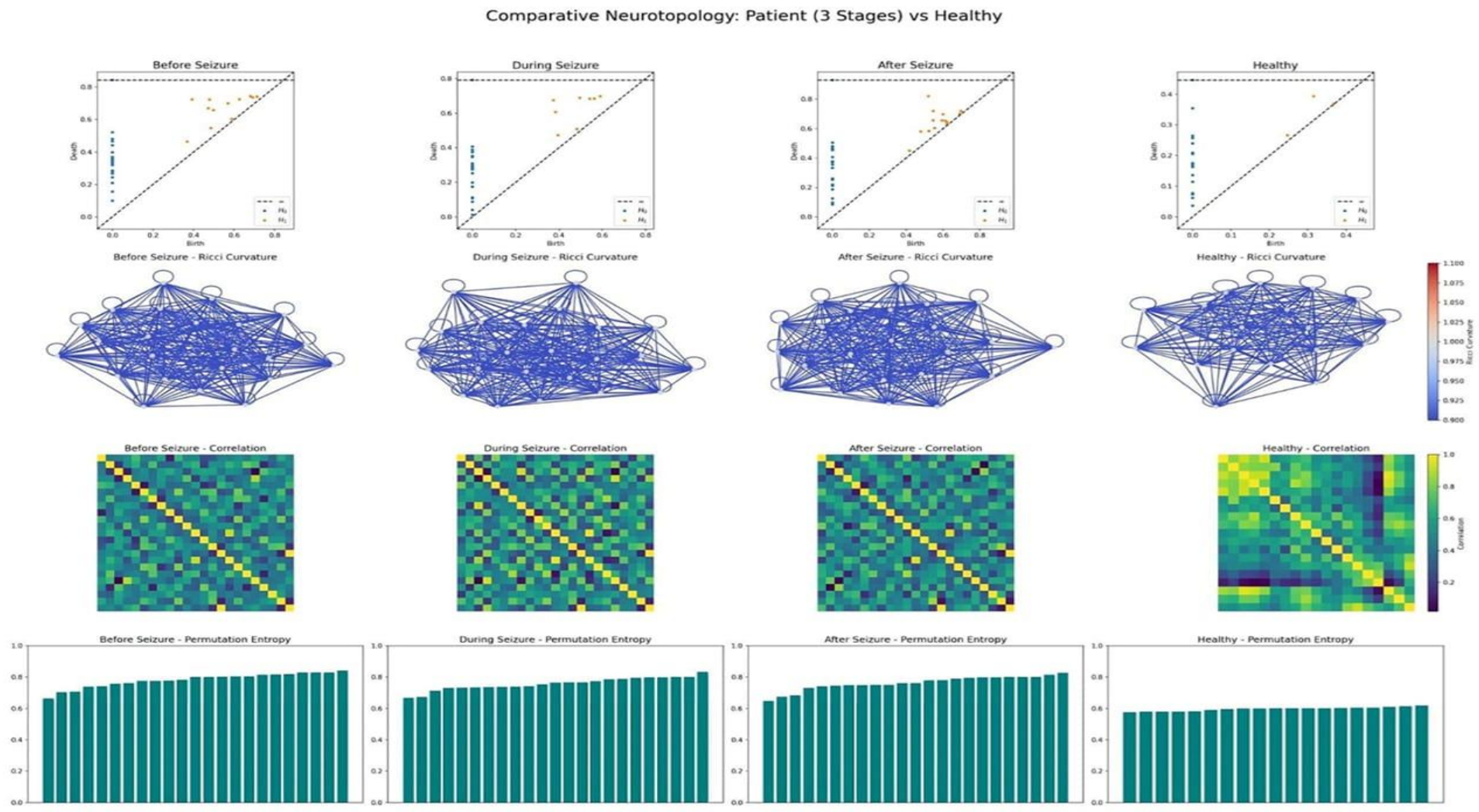

Figure 8 *This dashboard contrasts the key metrics (Homology, Ricci curvature, Correlation and Entropy) of the patient in its three phases against a Healthy control. A significant baseline difference is observed: the Healthy state shows a coherent and stable structure. Instead, the patient exhibits altered dynamics culminating in a collapse of network structure and functional complexity during the ictal phase (**during**), confirming its pathological neurotopological signature.*

Table 3 details the specific mean values for the key metrics compared in **Fig 7**. It provides a scientific interpretation of the differences between the Epileptic Patient and Healthy Control. While the healthy brain maintains a complex "isostatic" geometry (characterized by high Kurtosis and long-range memory), the epileptic patient exhibits a "collapsed" signal characterized by low complexity, loss of hierarchy, and degraded functional integration.

We propose a new terminology, derived from these findings, to describe this critical distinction. We define the neurotopological signature of healthy control characterized by low ordered complexity (a coherent, flexible, and positively curved architecture)—as isostasis. This state should not be confused with simple inactivity; It represents a geometry of maximum efficiency, stability and vital persistence.

In direct contrast, we define the signature of ictal crisis—characterized by disordered low complexity (a rigid, geometrically broken, negatively curved architecture)—as pathological metastasis.

Thus, the paradox is resolved: epileptic collapse is not an "explosion" of chaos, but the annihilation of isostatic geometry. This refutes the notion that "less entropy equals more health," proving that the key is not the *quantity* of information, but the *quality of the geometry*.

The clinical implications of this distinction (isostasis statistical analysis ($p < 0.001$) distinguishes the epileptic patient from the healthy patient even in the basal phases (pre- and post-ictal) suggests that this framework detects a *loss of baseline isostatic capacity*, not just the seizure. This establishes geometric analysis as a superior biomarker for neurodynamic pathology.

5. Conclusion

In conclusion, this work empirically demonstrates that epileptic pathology is not a high-entropy event, as traditional metrics suggest, but a *collapse of the geometry* of the neurodynamic network. Our results ($p < 0.001$) validate a statistically significant neurotopological distinction between status epilepticus (both baseline and ictal) and healthy control.

The central finding is the formal differentiation, made possible by this multi-metric framework, between the "low ordered complexity" of healthy rest (isostasis) and the "low disordered complexity" (rigidly and geometrically broken) of the pathological crisis. We thus prove that the key to distinguishing health from pathology does not lie in the *quantity* of information (entropy), but in the *quality of its geometry*. This framework establishes geometric analysis—particularly Ricci curvature and persistent homology—as a superior and most accurate neurodynamic biomarker for the characterization of epilepsy.

Acknowledgement

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Data and code availability statement

The EEG data from epileptic patients used in this study are publicly available and were obtained from the CHB-MIT database on PhysioNet (Goldberger et al., 2000), accessible at:

https://physionet.org/content/chbmit/1.0.0/

The complete computational code developed by the authors for this analysis is publicly available. The code is organized into three GitHub notebooks:

Data Cleaning and Metric Computation
https://github.com/dhay-star/NeuroDynamics-Collapse-Validation-/blob/main/fbd-part-one.ipynb

Epileptic Patient Analysis
https://github.com/dhay-star/NeuroDynamics-Collapse-Validation-/blob/main/fbd-part-three.ipynb

Healthy vs. Ictal Comparative Analysis
https://github.com/dhay-star/NeuroDynamics-Collapse-Validation-/blob/main/fbd-part-two.ipynb

Conflict of Interest

The authors declare no conflict of interest.

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